

NAG Toolbox for MATLAB

f07mg

1 Purpose

f07mg estimates the condition number of a real symmetric indefinite matrix A , where A has been factorized by f07md.

2 Syntax

```
[rcond, info] = f07mg(uplo, a, ipiv, anorm, 'n', n)
```

3 Description

f07mg estimates the condition number (in the 1-norm) of a real symmetric indefinite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – **string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^T P^T$, where U is upper triangular.

uplo = 'L'

$A = PLDL^T P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*)** – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

Details of the factorization of A , as returned by f07md.

3: **ipiv(*)** – **int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D , as returned by f07md.

4: **anorm** – double scalar

The 1-norm of the **original** matrix A , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07md or else from a copy of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The second dimension of the array **a** The dimension of the array **ipiv**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, iwork

5.4 Output Parameters1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **ipiv**, 6: **anorm**, 7: **rcond**, 8: **work**, 9: **iwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07mg involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to f07me with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogues of this function are f07mu for Hermitian matrices and f07nu for symmetric matrices.

9 Example

```
uplo = 'L';
a = [2.07, 0, 0, 0;
     4.2, 1.15, 0, 0;
     0.2230413840558341, 0.8115010321439103, -2.59067708640519, 0;
     0.6536583767489105, -0.5959697237786296, 0.3030846795506181,
     0.4073851981348882];
ipiv = [int32(-3);
        int32(-3);
        int32(3);
        int32(4)];
anorm = 11.29;
[rcond, info] = f07mg(uplo, a, ipiv, anorm)

rcond =
    0.0132
info =
    0
```
